LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – NOVEMBER 2007

BB 6

ST 3809 - STOCHASTIC PROCESSES

Date : 26/10/2007 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

Section-A(10×2=20 marks)

Answer ALL the questions. Each question carries TWO marks

1. Define a discrete time Markov Chain with stationary transition probabilities.
2. Classify the states of the following Markov chain with state space{1,2} having the following transition probability matrix.

$$P = \begin{bmatrix} 1 & 0 \\ 0.8 & 0.2 \end{bmatrix}$$

3. Let j be a state for which $f_{jj}^{(n)} = n/(2^{n+1})$, n>0. Show that j is recurrent.
4. If $\lim_{n\to\infty} p_{jj}^{(n)} > 0$, show that state j is aperiodic.
5. Write down the postulates of Poisson process.
6. Under the condition X(0)=1, obtain the mean of Yule process.
7. Define renewal function and find the same, when the inter occurrence times are independent and identically distributed exponential.
8. Define Markov Renewal process.
9. Consider a Branching process $\{X_n, n=0,1,2,\ldots\}$ with the initial population size $X_0=1$ and the following off-spring distribution.

   $p_0=1/8$, $p_1=1/2$, $p_2=1/4$, $p_3=1/8$.

   Find the mean of the population size of the n^{th} generation.

10. Distinguish between wide-sense and strictly stationary processes.

Section-B (5×8=40 marks)

Answer any FIVE questions. Each question carries EIGHT marks.

11. Consider a Markov chain on the non-negative integers such that, starting from state j, the particle goes in one step to the state (j+1) with probability p or to the state 0 with probability (1-p).
    i. Show that the chain is irreducible
    ii. Find $f_{00}^{(n)}$, n≥1.
    iii. Show that the chain is recurrent.

12. Define the period of a state in a Markov chain. Show that states belonging to the same class have the same period.

13. Obtain the system of differential equations satisfied by the transition probabilities of Yule process and calculate its transition probabilities with the initial condition X(0)=N.

14. Define compound Poisson process. For this process, obtain the generating function, expectation and Variance.

15. What are the postulates governing the birth and death process? Obtain the differential –difference equations governing this process.

16. Determine E[N(t)] and Var{N(t)}, where $\{N\{t\} \mid t \in [0,\infty)\}$ is a Renewal process.

17. Define (i) Submartingale and (ii) Super martingale. If $\{X_n\}$ is a Martingale with respect to {Yn}, show that $E[X_{n+k} \mid Y_0,Y_1,\ldots,Y_n]=X_n$, for all k=0,1,2,3,……

18. Derive the recurrence relation satisfied by the probability generating function, where $\{X_n, n=0,1,2,\ldots\}$ is a Branching process with $X_0=1$.

Section-C (2×20=40)

Answer any TWO equestions. Each question carries TWENTY marks.

19. a. State and prove Chapman - Kolmogorov equations for a discrete time Markov chain. (10 marks).

b. Consider a random walk on the integers such that $p_{i,\,i+1} = p$, $p_{i,i-1}=q$ for all integers i (0<p<1, p+q=1). Determine $p_{00}^{(n)}$. (10 marks)

20. a. State and prove the Basic limit theorem of Markov chains.(12 marks)

b. Examine whether stationary distribution exists for the Markov chain with transition probability matrix

$$P = \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.6 & 0.0 & 0.4 \\ 0.0 & 0.4 & 0.6 \end{bmatrix}$$

.Obtain the same if it exists. (8 marks)

21. a. For a Poisson process $\{X\{t\}, t \in [0,\infty)\}$,show that

$$P[X(t)=n] = e^{-\lambda t}(\lambda t)^n/n!\ ,\ n=0, 1, 2,\ldots$$ (10 marks)

b. The number of accidents in a town follows a Poisson process with the mean of 2 accidents per day and the number of people involved in i^{th} accident has the distribution $P[X_1=k] = 1/2^k$, k=1,2,3,…….. Find the mean and variance of the number of people involved in accidents per week. (10 marks)

22. a. Let $\{X_n, n\geq 0\}$ be a Branching process with one ancestor. Let the off-spring distribution be $p_j=bc^{j-1}$, b>0, c>0, b+c<1, j=1,2,3,…; $\sum_{j=0}^{\infty} p_j=1$. Obtain E[Xn], Var(Xn) and $cov(X_m,X_n)$, m<n. (12 marks)

b. State and prove the prediction theorem for the minimum mean square error predictors. (8 marks)

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